

Superconductivity in a two-dimensional Electron Gas

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In a series of recent experiments, Kravchenko and colleagues^{1,2} observed unexpectedly that a two-dimensional electron gas in zero magnetic field can be a conductor. The two-dimensionality was imposed by confining the electron gas to move laterally at the interface between two semiconductors. The observation of a conductor in two dimensions (2D) is surprising as the conventional theory of metals precludes the presence of a metallic state at zero temperature in 2D³. Nonetheless, there are now several experiments confirming the existence of the new conducting phase in a dilute two-dimensional electron gas in zero magnetic field⁴⁻⁷. Here we argue based on an analysis of the experiments and general theoretical grounds that this phase is a zero-temperature superconductor with an inhomogeneous charge density.

While the specific details differ as the semiconductor that confines the 2D electron gas is changed, several key similarities^{1,2,4-7} exist among the reported observations of the transition to the conducting state: 1) the existence of a critical electron or hole density, n_c , above which the conducting phase appears, 2) a characteristic temperature, typically on the order of half the Fermi temperature, T_F , below which the resistivity on the conducting side decreases, 3) critical scaling, indicative of a quantum critical point, in the vicinity of the insulator-conducting phase transition, 4) non-linear current-voltage (I-V) curves that exhibit a symmetry around the I-V curve at criticality, and 5) suppression of the conducting phase by a magnetic field. In Si metal-oxide-field-effect-transistors (MOSFET's) samples¹, temperature (T) and electric field (E) scaling have made it possible to extract the dynamical and correlation length exponents, z and ν , respectively. By fitting the resistivity measurements to functions of the form, $\rho(T, n) = f_1(|\delta|/T^b)$ and $\rho(E, n) = f_2(|\delta|/E^a)$, with n , the electron density, $b = 1/z\nu$, $a = 1/[(z+1)\nu]$, and $\delta = (n - n_c)/n_c$, Kravchenko and colleagues¹ found that $\nu = 1.5$ and $z = 0.8$. Analogous scaling occurs in GaAs⁵ but only in the most disordered samples. This indicates that the underlying transition in the clean system is first order, whereas in the presence of disorder, it becomes second order and critical scaling applies.

Three postulates anchor the conventional theory of metals³: 1) Fermi liquid theory accurately describes the low-temperature physics of conventional clean metals, 2) the classical and quantum corrections to the conductivity

are additive⁸ as the system size increases, and 3) the logarithmic derivative (β) of the dimensionless conductance with respect to the system size is a continuous monotonic single-valued function as the strength of the disorder is increased from weak to strong. From (2) and (3) it follows^{3,8} that β is always negative in two dimensions. Hence, as the system size increases, the conductance decreases and insulating behavior necessarily obtains in two dimensions. The general applicability of the scaling analysis rests firmly on the extensive numerical and experimental work that have confirmed its central predictions and assumptions⁹.

Strictly speaking, however, Fermi liquid behaviour obtains only for $T \ll T_F$. Hence, the onset of a conducting phase at $T \approx T_F$ does not necessarily pose a paradox provided that the resistivity turns upwards at sufficiently low temperatures. However, such behaviour is inconsistent with the experimental observations for two primary reasons. First, there is no experimental indication that the resistivity turns upwards at low temperatures. Secondly, the critical scaling observed¹ signifies that the conductor-insulator transition is a bona-fide quantum phase transition.

The conventional theory of metals forces us then to come to grips with the new conducting state in 2D by abandoning one or all of the postulates of the standard view. In the context of the experiments^{1,2,4-7}, the Coulomb interaction V_{ee} exceeds the Fermi energy, ϵ_F , by at least an order of magnitude. For long-range interactions, non-Fermi liquid behaviour is likely to obtain when $V_{ee} \gg \epsilon_F$. It is for this reason and the localization constraint placed on Fermi liquids in 2D by the conventional theory that the experiments force us to consider non-Fermi liquid scenarios only. In this vein, Chakravarty and co-workers¹⁰ proposed a Luttinger liquid state to explain the experimental observations. However, such a state has yet to be proven to exist except in 1-dimension. In fact in 2D, a $T = 0$ superconductor is the only conducting non-Fermi liquid state proven to exist in the presence of disorder¹¹ and zero magnetic field. Hence, rather than speculate as to the existence of some yet-unproven non-Fermi liquid state in 2D, we take the conservative approach and propose that superconducting fluctuations mediate the new conducting state in 2D.

We support this proposal with three subsidiary arguments: 1) the generic features¹² of the conducting transition resemble those of known¹³ insulator-superconductor phase transitions, 2) the magnetoresistance measurements^{1,5} provide evidence for the existence

of a critical magnetic field above which the conducting phase is destroyed, and 3) the conducting transition lies in close proximity to an incipient electron crystal state in which strong charge retardation effects can lead to Cooper pair formation. Further, we draw a parallel between the 2D electron gas experiments and those on inhomogeneous superconductors in which an apparent saturation of the resistivity has been observed at low temperatures^{14,15}. We argue that the enhanced role of classical fluctuations of the superconducting phase¹⁶ in 2D can significantly suppress the temperature at which the resistivity vanishes.

In support of the superconducting scenario are numerous transport measurements on the electron and hole systems. The value of the dynamical exponent¹³, $z = 1$, the non-linearity in the I-V curves as well as the observed temperature and electric field scaling of the resistivity are all consistent with what is observed in known insulator-superconductor transitions (IST's). We now analyze the experimental data on the magneto-resistance and show that they offer evidence for a critical parallel magnetic field, consistent with singlet pairing. Although we limit our discussion to a parallel field ($H_{||}$), Kravchenko and colleagues² showed that the response of the conducting phase to a magnetic field is independent of the direction of the field, indicating that it is a spin effect that destroys the conducting phase. Consider first the magneto-resistance measurements in GaAs⁵. Measurements of the conductivity as a function of the hole density in GaAs in the temperature interval [1.4K, 0.3K] reveal that even in the presence of a magnetic field, the conductivity curves cross at a unique value of the hole density. The single crossing point signifies that the conductivity is temperature independent at a particular density. Hence, this density demarcates the transition between the conducting and insulating phases. If the conducting phase were destroyed by an arbitrarily small magnetic field, such a crossing point would not occur at finite field. However, the experiments show that even for fields as high as $1T$, a unique crossing point exists. By $H_{||} = 3.0T$, the unique crossing point vanishes, indicating that there is a threshold parallel magnetic field above which the conducting phase is extinguished. The critical field is, however, density dependent. From the crossing point in finite field, we conclude that for $\delta = 0.02$ and $\delta = 0.03$, the critical fields are $H_{||}^c = 0.5T$ and $H_{||}^c = 1.0T$, respectively. For GaAs, the Zeeman energies at such field strengths correspond to an energy at least an order of magnitude smaller than the Fermi temperature. Hence, the conducting phase in GaAs is characterized by an internal energy scale distinct from ϵ_F .

What about Si MOSFET's? Based on the measurements of $\rho(H, E)$, Kravchenko, et. al.¹ concluded that the conducting phase is destroyed for an arbitrarily small parallel magnetic field, indicating a vanishing of the critical field. This conclusion poses a distinct problem if the conducting transition in Si MOSFET's and GaAs is

assumed to be driven by the same physics. That is, either both or neither should display a critical field, H_c . We point out, however, that the observation of a critical field in GaAs was based on an analysis of $\rho(H, T)$, not $\rho(H, E)$. If this conclusion is correct, $\rho(H, T)$ should also confirm this result. Hence, we investigate precisely what is contained in $\rho(H, T)$ for Si MOSFET's. The raw data for Si MOSFET's indicates that for $H_{||} \approx 9kOe$ the slope of $\rho(T)$ changes sign². Hence, from the raw data, there is no indication at least within the temperature regime studied that an arbitrarily small magnetic field suppresses the conducting phase. As a consequence, we analyzed $\rho(H, T)$ for a Si MOSFET (D. Simonian, S. Kravchenko, and M. Sarachik, personal communication) with a scaling function of the form, $f(|H_{||} - H_c|/T^{1/\alpha})$. In field-tuned 2D IST's^{13,17}, the resistivity scales on either side of H_c as a universal function¹⁷ of $|H - H_c|/T^{1/z_B\nu_B}$, where z_B and ν_B are the dynamical and correlation length exponents, respectively in a magnetic field. If a critical field exists, then above and below H_c the experimental values for the resistivity should collapse onto two distinct branches. Such collapse is shown in Fig. (1) above and below a critical field of $H_c = 9.5kOe$ with $\alpha = 0.6 \pm 0.1$. Here again, the critical field corresponds to an energy scale that is an order of magnitude smaller than the Fermi energy. It is interesting to note that experiments on the IST in bismuth films (N. Markovic, Christiansen, and A. M. Goldman, personal communication) show that $z_B\nu_B = 0.7 \pm 0.2$, which is remarkably close to the value, $\alpha = 0.6 \pm 0.1$, obtained in the scaling plot in Fig. (1).

The stiffness in the conducting phase to a parallel magnetic field signifies that the ground state is a singlet. While spin glass and antiferromagnetic order are also consistent with a singlet ground state, such phases are insulating in 2D. If we entertain the possibility of other non-Fermi liquid states, it is unclear how such a state will differ from a superconducting one, as the experiments dictate that such a state must also have a singlet energy gap and conduct in the presence of disorder.

In terms of the dimensionless measure of the density, $r_s = 1/\sqrt{\pi}pa_0^*$, a_0^* the effective Bohr radius, the conducting transition for Si MOSFET's¹ and GaAs^{5,6} occurs at $r_s \approx 10$ and $r_s \approx 18$, respectively. For such dilute systems, $r_s \gg 1$, it is not known definitively what type of microscopic order obtains in the ground state. However, as a Fermi liquid description is valid for dense systems, $r_s \lesssim 1$, perturbation theory^{18,19} will necessarily fail to describe the experimental observations^{4,20}. Monte Carlo simulations²¹ reveal that in a clean 2D electron gas, a Wigner crystal is the ground state for $r_s > 37$. A Wigner crystal is an electron crystalline phase in which the electrons minimize their repulsive potential energy and form an ordered array. For clean 2D systems, the electrons arrange themselves in a triangular lattice. The simulations also indicate that in the presence of disorder^{22,23} or random pinning centers, an incipient Wigner crystal

phase exhibiting quasi-long range order still forms. The melting density for such a phase is shifted to higher values, typically $r_s \approx 10$ (though the amount and type of disorder might change this value) because disorder stabilizes the solid relative to the liquid phase. As disorder is undoubtedly present in the experimental systems and the conducting transition occurs for $r_s \approx 10$, it makes sense to think about the transition to the conducting state as a transition from an insulating incipient Wigner crystal phase. Experimentally, the density dependence of the threshold electric field²⁴ required to initiate transport on the insulating side, $E_t \propto \delta^{1.5}$, is consistent with the prediction²⁵ for quantum tunneling in an incipient Wigner crystal. Hence, experimental evidence also corroborates the proximity of the conducting transition to an insulating phase with quasi-long range order.

The proximity of the conducting phase to a quasi-crystalline dilute electron phase makes this problem quite analogous to high temperature superconductivity (high T_c). While this phenomenon is currently unexplained, experimentally it is clear that superconductivity in the copper oxides obtains as doped holes disrupt the perfect antiferromagnetic order in the Mott insulating phase. The correspondence with high T_c is even more striking as such systems are quasi-2D with $r_s \sim 10$ for the holes. In both systems, destruction of the long-range or quasi-long range spin or charge order in the insulating phase is accompanied by a charge or spin retardation effect which attempts to preserve the memory of the correlations in the insulating phase. As an incipient Wigner crystal has at best frustrated antiferromagnetic²¹ spin order, the charge retardation effect is expected to dominate²⁶. We illustrate its role as follows.

Consider an electron crystalline or quasi-crystalline phase in which the electrons are locked into ‘home’ positions by the electron interaction. Such a crystalline phase is stable when the zero-point energy $\hbar\omega_0$ much exceeds the kinetic energy, ϵ_F , of an electron in each unit cell (or Wigner-Seitz cell in the context of a Wigner crystal) of edge length r_s . A distinct feature of an electron crystal or quasi-crystalline phase is the dominant role played by the correlations. The correlations manifest themselves in the form of a correlation hole which is centered at the average position of each electron in a unit cell. Increasing the electron density leads to an increase in the electron kinetic energy and an eventual melting of a Wigner crystal. In the melted phase, correlation holes still form around electrons. However, because they are massive, relative to a single electron, their response is delayed. The retardation timescale is set by the inverse of the plasma frequency. Within this timescale, a correlation hole lags behind its associated electron and hence appears positively charged to another electron. As a consequence, the partially-vacated correlation hole can attract another electron. In so doing, the correlation hole mediates a dynamic attraction between electrons and the subsequent onset of Cooper pair formation. Because plasmons are ungapped in 2D, the retardation effect is strongest for

the low frequency plasmons. We anticipate that it is from such plasmons that the dominant electron attraction arises.

For a 3D electron gas, Takada^{26,28} has made the analogous observation regarding the proximity of superconductivity to the melting of a Wigner crystal. The work of Kelly and Hanke²⁷ and Ren and Zhang²⁸ also relates to this proximity. More recently, Belitz and Kirkpatrick²⁹ have proposed a similar charge polarization mechanism in which the long-time tail of the charge density correlation function, which appears in the presence of disorder, assists pairing. Also, in the Monte Carlo simulations²¹ on a clean Wigner crystal, a precipitous drop of the spin susceptibility is observed in the melted phase, a pre-requisite for singlet superconductivity. In addition, capacitance charging experiments on GaAs quantum dots³⁰, in which a 2D electron gas is constructed one electron at a time from the very first electron, have reported the occurrence of pair electron charging events in the density range $2.8 < r_s < 9.6$. As pair-tunneling events correspond to two electrons charging the same quantum state on the dot, such states form only if an electronic attraction screens the Coulomb repulsion between the two electrons. In so far as the electrons in a quantum dot realistically model a dilute ($r_s > 3$) 2D electron gas, it is certainly reasonable to suspect that the electron attraction persists in Si MOSFET’s as well.

In 2D superconductors, the resistivity vanishes not at the mean-field temperature, T_c , at which the pair amplitude is established but at a lower temperature, T_θ , where global phase coherence obtains. That is, rather than defining the superconducting transition temperature, T_c determines the characteristic temperature at which the resistivity initially decreases. Both disorder³¹ and low superfluid densities¹⁶ can significantly lower T_θ relative to T_c in 2D systems. In fact, Emery and Kivelson¹⁶ have argued that a key feature which suppresses T_θ in high T_c materials is their notoriously low superfluid density. In the electron systems of interest, disorder is present (leading to inhomogeneous electron density) and the superfluid density is expected to be small as the electron gas is dilute. As a consequence, we anticipate that in Si MOSFET’s and GaAs, T_θ , will be greatly suppressed relative to the mean-field T_c . In the case of GaAs,^{6,5} exponential drop of the resistivity on the conducting side is accompanied by a plateau below some characteristic temperature. This latter experimental trend is identical to the behaviour observed by Goldman and collaborators¹⁴ and Imry and Strongin¹⁵ in inhomogeneous 2D superconductors. The strong similarity between the inhomogeneous systems and the GaAs data suggests that the phase-locked zero-resistance state occurs at a temperature significantly lower than that probed experimentally. Nonetheless, the non-linear I-V characteristics¹ observed in the Si MOSFET’s is consistent with a 2D superconducting state on the brink of global phase coherence. However, as long as $T > T_\theta$, pair fluctuations are quenched in a magnetic field only when the Zeeman

and pairing energies are comparable.

In the dense limit, correlation and retardation effects are negligible and consequently, Cooper pair formation ceases. The anticipated termination of the electron attraction at small r_s suggests the schematic phase diagram depicted in Fig. (2). Beyond the upper density at which superconducting pair fluctuations cease, the electron liquid is most likely insulating. Hence, a direct transition from an insulator to a metal in 2D by changing the density seems unlikely. In GaAs samples, a re-entrant (M. Y. Simmons, personal communication) insulating phase at $r_s \approx 8$ was observed consistent with the analysis here. Further experiments are needed on the MOSFET's to see the re-entrant insulating phase.

In closing, because Fermi liquids are localized in 2D, the new conducting state must be some type of non-Fermi liquid. As $T = 0$ superconductivity is the only conducting, disordered non-Fermi liquid proven to exist in 2D, this must be viewed as the leading candidate to explain the experimental observations. The existence of a critical parallel magnetic field is a clear indicator of a singlet energy gap in the conducting phase in 2D. However, further experiments are needed to probe the low temperature physics as well as the magneto-resistance in Si MOSFET's. In addition to disorder and a suppressed superfluid density, the inhomogeneities introduced in a dilute electron gas ($r_s > 3$) as a result of the negative compressibility³² (an instability relative to a uniform charge density) are expected to enhance phase fluctuations as well. Consequently, we anticipate that the transition to the phase-locked state will occur at a temperature significantly below the mean-field T_c .

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FIG. 1. Scaling curve for the magnetoresistance obtained from the experimental data of Simonian, Kravchenko, and Sarachik illustrating clearly the existence of a critical field at $9.5kOe$ with $\alpha = 0.6$.

FIG. 2. Schematic mean-field phase diagram for a 2D disordered electron system. DWC=Disordered Wigner crystal and SC=superconductor. The black circle at n_c is determined from the experimental data in Ref. 1

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Field-tuned scaling for resistivity, $H_c \approx 9.5$ kOe



